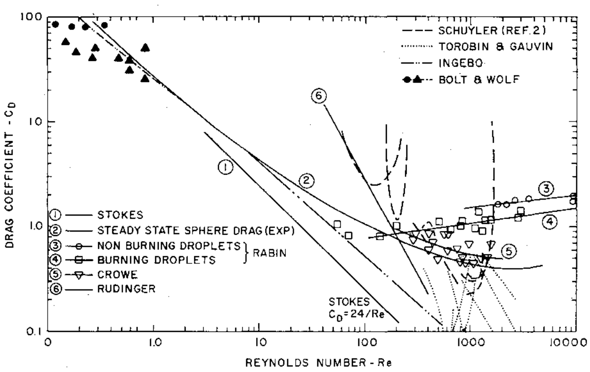

Fig. 1 Small particle drag coefficient vs Reynolds number.

are superimposed on Fig. 2 of the author's paper and presented here as Fig. 1. Also shown is Stoke's law.

The range of Reynolds number studied was 500–1500. Each of the curves of the Schuyler data in Fig. 1 represents the history of a single droplet. Each curve starts at a low *Re* and goes toward a large *Re*. This is due to the air (and also relative) velocity increasing faster than the decrease of the droplet's diameter. It is remembered that the air velocity was designed to increase linearly with distance.

Perhaps the largest droplet measurement error was that associated with its diameter. Initially the droplets are relatively spherical, but as they proceed down the chamber (Fig. 2), the dynamic pressure force flattens them, creating a large area perpendicular to the air flow. However, in the photographs only their side area could be seen. This means that the actual drag coefficient should be larger than that recorded in Fig. 1. This is particularly true for the higher Reynolds number (larger diameter) droplets. The smaller droplets did not appear to flatten as much. In Fig. 2 can be seen a relatively spherical droplet, a highly flattened droplet, and a droplet which has been distended into a cup shape whose stretched membrane is just about to rupture.

The present data, without correction for droplet diameter, correlate approximately with Rudinger's glass bead results. If, however, a more appropriate diameter (associated with the frontal area) could be measured, the present high *Re* data would be corrected probably upward and most likely would fall more in line with Rabin's data.

Reported in Ref. 2 is the possibility that for a given Reynolds number, the drag coefficient decreases with acceleration. The data of Ref. 2 may be put in three acceleration groups. For $Re \approx 150$, $\alpha \approx 80$ ft/sec^2; $Re \approx 400$, $\alpha \approx 32$ ft/sec^2; and $Re \approx 1000$, $\alpha \approx 200$ ft/sec^2. The data of Ingebo in Fig. 2 have an acceleration of approximately 10,000 ft/sec^2. Calculating the acceleration modulus for Schuyler's data yields $Ac \approx 10^{-4}$. According to Crowe,[3] this should indicate that the droplet's drag coefficient should be negligibly effected by acceleration. For the present drag data, however, this does not appear to be the case. If the data were corrected for frontal area, however, acceleration effects might drop out.

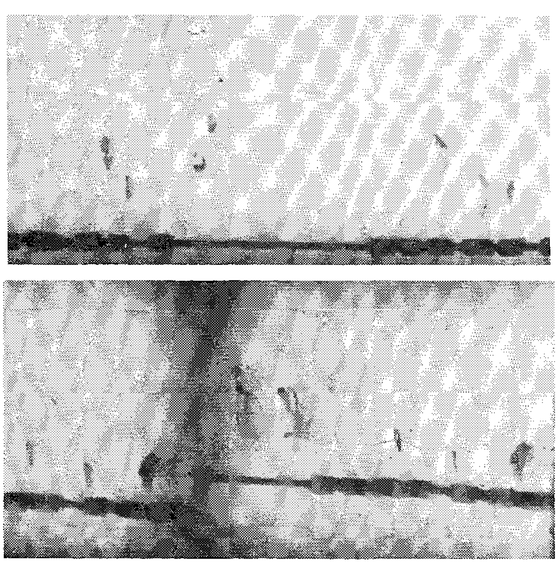

Fig. 2 Photograph of liquid droplet (Freon -12, CCl_2F_2) deformation in a linearly increasing air velocity field (two successive frames, the first one on top).

The relative Mach number M_R for the data of Ref. 2 was approximately 0.05. Therefore, the relative Mach number effect referred to in the authors paper should be of no concern in the interpretation of Schuyler's data. Because higher values of M_R were not studied by Schuyler, no corroboration of the Mach number effect is possible.

In conclusion, it can be stated that the authors contention that small particle drag coefficients can be expected to be substantially higher for rocket combustion chamber conditions than quiescent conditions, appears to be further substantiated by the data of Ref. 2. Though proper reduction of the higher *Re* data of Ref. 2 was hampered by the inability to measure droplet frontal area, such a correction to the data would yield even higher values of drag coefficient.

It is agreed that even in light of past investigations, a substantial amount of work still is required to determine the effects of compressibility, of low Reynolds number, burning (mass loss), neighboring particles, and turbulence on small particle trajectories. For deformable particles, photographic data on frontal area variations must be obtained also.

References

[1] Selberg, B. P. and Nicholls, J. A., "Drag Coefficient of Small Spherical Particles," *AIAA Journal*, Vol. 6, No. 3, March 1968.

[2] Schuyler, F. L., "Combustion Instability: Liquid Stream and Droplet Behavior Part 1. Experimental and Theoretical Analysis of Evaporating Droplets," TR 59–720, Sept. 1960, Wright Air Development Div.

[3] Crowe, C. T., Nicholls, J. A., and Morrison, J. B., "Drag Coefficients of Inlet and Burning Particles Accelerating in Gas Streams," *Ninth Symposium on Combustion,* Academic Press, New York, 1963, pp. 395–406.

Reply by Author to F. L. Schuyler

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IN the Comment immediately preceding this, Schuyler refers to the work of Selberg and Nicholls[1] and presents some of his experimental results on the drag and acceleration of evaporating liquid drops. On the basis of these results, Schuyler poses the possibility that acceleration effects are important in his drag data. I seriously doubt this, for, as he has indicated, there are other effects involved in the acceleration of the liquid drops. If one calculates the range of Weber and Reynolds numbers for the experiments, it is apparent that the drops must undergo a transition from a bag to a stripping mode of disintegration. Ranger and Nicholls[2] have presented results for the stripping mode that indicate that the drop diameter increases to more than 3 times the initial diameter in the transverse direction. The windward face takes on an elliptical profile and the leeward face, while usually not visible, probably takes on a concave shape. At any rate, the drop distorts to a shape far from that of a

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sphere. Ranger also showed that the drag coefficient based on initial diameter and for a wide range of conditions is about 3.0, a value in reasonable agreement with that of Rabin. In view of this, it is believed that the unusual C_D-Re curve shown in Schuyler's Fig. 1 is really a masked diameter or distortion effect and that C_D does not change that much. Accordingly, there appears to be no evidence to support a discernible effect of acceleration on C_D when the acceleration modulus is much less than unity. Selberg attributed his high values of C_D to surface roughness. This was further substantiated in a follow-on investigation by Sivier,[3] who used magnetically levitated sphere in a subsonic wind tunnel. Further, the results of Rudinger and Ingebo can not be taken as a case for an acceleration effect in that they used clouds of solid and liquid particles, respectively, with the attendant complex and unknown flowfields.

References

[1] Selberg, B. P. and Nicholls, J. A., "Drag Coefficient of Small Spherical Particles," *AIAA Journal*, Vol. 6, No. 3, March 1968.

[2] Ranger, A. A. and Nicholls, J. A., "Aerodynamic Shattering of Liquid Drops," Paper 68-83, 1968, AIAA; also *AIAA Journal* Vol. 7, No. 2, Feb. 1969, pp. 285–290.

[3] Sivier, K. R., "Subsonic Sphere Drag Measurements at Intermediate Reynolds Numbers," Ph.D. thesis, Univ. of Michigan, 1967.

Comment on "Solution of a Three-Dimensional Boundary-Layer Flow with Separation"

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In Ref. 1 a numerical solution is given for the equations of motion of three-dimensional laminar incompressible boundary layers. The method is restricted to non-negative tangential velocity components u and v. It is the purpose of this Comment to remove this restriction and show that numerical solutions can also be obtained for negative tangential velocity components.

For two-dimensional boundary layers, the stability of the difference equations can always be guaranteed by writing them implicitly for the direction normal to the wall. For three-dimensional boundary layers, the corresponding difference equations are only conditionally stable, even when they are written in an implicit form. This can be explained by the appearance of the term $v\partial/\partial y$ in the convective operator of the momentum equations (y denotes the second tangential co-ordinate). Consequently, there are several ways in which the difference quotients for the direction normal to the wall can be incorporated in the difference equations. In Ref. 1 $\Delta^2 u/(\Delta z)^2, \Delta u/\Delta z$, etc. are formed at the four corner points of a mesh cell in the x-y plane. By determining the amplification factor[2] for the linearized differential equations, one can show that the resulting difference equations are stable for

$$0 \leq v\Delta x/u\Delta y \tag{1}$$

The validity of this condition is confirmed by the numerical results of Ref. 1. It seems worthwhile to point out that the difference quotients for the direction normal to the wall need only be formed at two diagonally opposite points of the mesh cell. The truncation error and the stability condition remain unchanged; however, the computational effort is very much reduced. It is understood that one of the two points in question must always be a point for which u and v are to be calculated.

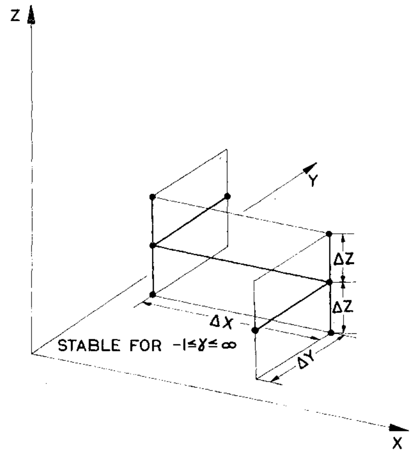

Fig. 1 Difference scheme for three-dimensional boundary layers.

Another way of combining the difference quotients[3] is indicated in Fig. 1. That difference scheme has the same truncation error as the one given in Ref. 1. Its amplification factor is

$$G(\Delta x) = \frac{2 + \gamma(1 - \cos k_2\Delta y) + \psi - i\gamma \sin k_2\Delta y}{2 + \gamma(1 - \cos k_2\Delta y) - \psi + i\gamma \sin k_2\Delta y} \tag{2}$$

The quantities γ and ψ stand for the following expressions:

$$\gamma = v\Delta x/u\Delta y \tag{3}$$

$$\psi = -\frac{2\mu\Delta x}{\rho u \Delta z^2}(1 - \cos k_1\Delta z) \tag{4}$$

For the difference equations to be stable, one must require that the absolute value of the amplification factor be less than or equal to unity.[2] This condition yields the following result:

$$-1 \leq v\Delta x/u\Delta y \tag{5}$$

If the increments Δx and Δy are taken to be positive, then u or v may become negative,† and the difference equations

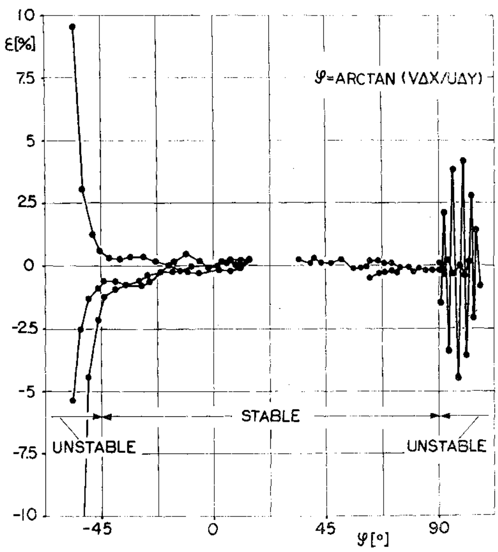

Fig. 2 Error of the shearing stress at the wall for the stable and unstable region of the difference scheme in Fig. 1.

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† Negative tangential velocity components should not be confused with a time reversal in the heat conduction equation. The time reversal is equivalent to a negative conductivity.